

16

Wave Motion

CHAPTER OUTLINE

- 16.1 Propagation of a Disturbance
- 16.2 Sinusoidal Waves
- 16.3 The Speed of Waves on Strings
- 16.4 Reflection and Transmission
- 16.5 Rate of Energy Transfer by Sinusoidal Waves on Strings
- 16.6 The Linear Wave Equation

ANSWERS TO QUESTIONS

- Q16.1** As the pulse moves down the string, the particles of the string itself move side to side. Since the medium—here, the string—moves perpendicular to the direction of wave propagation, the wave is transverse by definition.
- Q16.2** To use a slinky to create a longitudinal wave, pull a few coils back and release. For a transverse wave, jostle the end coil side to side.
- Q16.3** From $v = \sqrt{\frac{T}{\mu}}$, we must increase the tension by a factor of 4.
- Q16.4** It depends on from what the wave reflects. If reflecting from a less dense string, the reflected part of the wave will be right side up.
- Q16.5** Yes, among other things it depends on. $v_{\max} = \omega A = 2\pi f A = \frac{2\pi v A}{\lambda}$. Here v is the speed of the wave.
- Q16.6** Since the frequency is 3 cycles per second, the period is $\frac{1}{3}$ second = 333 ms.
- Q16.7** Amplitude is increased by a factor of $\sqrt{2}$. The wave speed does not change.
- Q16.8** The section of rope moves up and down in SHM. Its speed is always changing. The wave continues on with constant speed in one direction, setting further sections of the rope into up-and-down motion.
- Q16.9** Each element of the rope must support the weight of the rope below it. The tension increases with height. (It increases linearly, if the rope does not stretch.) Then the wave speed $v = \sqrt{\frac{T}{\mu}}$ increases with height.
- Q16.10** The difference is in the direction of motion of the elements of the medium. In longitudinal waves, the medium moves back and forth parallel to the direction of wave motion. In transverse waves, the medium moves perpendicular to the direction of wave motion.

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- Q16.11** Slower. Wave speed is inversely proportional to the square root of linear density.
- Q16.12** As the wave passes from the massive string to the less massive string, the wave speed will increase according to $v = \sqrt{\frac{T}{\mu}}$. The frequency will remain unchanged. Since $v = f\lambda$, the wavelength must increase.
- Q16.13** Higher tension makes wave speed higher. Greater linear density makes the wave move more slowly.
- Q16.14** The wave speed is independent of the maximum particle speed. The source determines the maximum particle speed, through its frequency and amplitude. The wave speed depends instead on properties of the medium.
- Q16.15** Longitudinal waves depend on the compressibility of the fluid for their propagation. Transverse waves require a restoring force in response to sheer strain. Fluids do not have the underlying structure to supply such a force. A fluid cannot support static sheer. A viscous fluid can temporarily be put under sheer, but the higher its viscosity the more quickly it converts input work into internal energy. A local vibration imposed on it is strongly damped, and not a source of wave propagation.
- Q16.16** Let $\Delta t = t_s - t_p$ represent the difference in arrival times of the two waves at a station at distance $d = v_s t_s = v_p t_p$ from the hypocenter. Then $d = \Delta t \left(\frac{1}{v_s} - \frac{1}{v_p} \right)^{-1}$. Knowing the distance from the first station places the hypocenter on a sphere around it. A measurement from a second station limits it to another sphere, which intersects with the first in a circle. Data from a third non-collinear station will generally limit the possibilities to a point.
- Q16.17** The speed of a wave on a “massless” string would be infinite!

SOLUTIONS TO PROBLEMS

Section 16.1 Propagation of a Disturbance

- P16.1** Replace x by $x - vt = x - 4.5t$

to get

$$y = \frac{6}{[(x - 4.5t)^2 + 3]}$$

P16.2

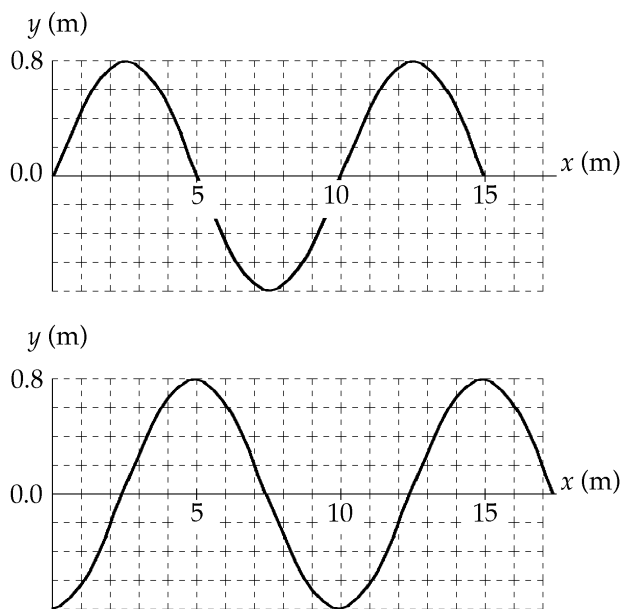


FIG. P16.2

P16.3 $5.00e^{-(x+5t)^2}$ is of the form $f(x + vt)$

so it describes a wave moving to the left at $v =$ 5.00 m/s.

P16.4 (a) The longitudinal wave travels a shorter distance and is moving faster, so it will arrive at point B first.

(b) The wave that travels through the Earth must travel

a distance of $2R \sin 30.0^\circ = 2(6.37 \times 10^6 \text{ m}) \sin 30.0^\circ = 6.37 \times 10^6 \text{ m}$

at a speed of 7 800 m/s

Therefore, it takes $\frac{6.37 \times 10^6 \text{ m}}{7\,800 \text{ m/s}} = 817 \text{ s}$

The wave that travels along the Earth's surface must travel

a distance of $s = R\theta = R\left(\frac{\pi}{3} \text{ rad}\right) = 6.67 \times 10^6 \text{ m}$

at a speed of 4 500 m/s

Therefore, it takes $\frac{6.67 \times 10^6}{4\,500} = 1\,482 \text{ s}$

The time difference is 665 s = 11.1 min

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P16.5 The distance the waves have traveled is $d = (7.80 \text{ km/s})t = (4.50 \text{ km/s})(t + 17.3 \text{ s})$

where t is the travel time for the faster wave.

Then, $(7.80 - 4.50)(\text{km/s})t = (4.50 \text{ km/s})(17.3 \text{ s})$

$$\text{or } t = \frac{(4.50 \text{ km/s})(17.3 \text{ s})}{(7.80 - 4.50) \text{ km/s}} = 23.6 \text{ s}$$

and the distance is $d = (7.80 \text{ km/s})(23.6 \text{ s}) = \boxed{184 \text{ km}}$.

Section 16.2 Sinusoidal Waves

P16.6 Using data from the observations, we have $\lambda = 1.20 \text{ m}$

$$\text{and } f = \frac{8.00}{12.0 \text{ s}}$$

$$\text{Therefore, } v = \lambda f = (1.20 \text{ m})\left(\frac{8.00}{12.0 \text{ s}}\right) = \boxed{0.800 \text{ m/s}}$$

$$\textbf{P16.7} \quad f = \frac{40.0 \text{ vibrations}}{30.0 \text{ s}} = \frac{4}{3} \text{ Hz} \qquad v = \frac{425 \text{ cm}}{10.0 \text{ s}} = 42.5 \text{ cm/s}$$

$$\lambda = \frac{v}{f} = \frac{42.5 \text{ cm/s}}{\frac{4}{3} \text{ Hz}} = 31.9 \text{ cm} = \boxed{0.319 \text{ m}}$$

$$\textbf{P16.8} \quad v = f\lambda = (4.00 \text{ Hz})(60.0 \text{ cm}) = 240 \text{ cm/s} = \boxed{2.40 \text{ m/s}}$$

$$\textbf{P16.9} \quad y = (0.0200 \text{ m})\sin(2.11x - 3.62t) \text{ in SI units} \qquad A = \boxed{2.00 \text{ cm}}$$

$$k = 2.11 \text{ rad/m} \qquad \lambda = \frac{2\pi}{k} = \boxed{2.98 \text{ m}}$$

$$\omega = 3.62 \text{ rad/s} \qquad f = \frac{\omega}{2\pi} = \boxed{0.576 \text{ Hz}}$$

$$v = f\lambda = \frac{\omega}{2\pi} \frac{2\pi}{k} = \frac{3.62}{2.11} = \boxed{1.72 \text{ m/s}}$$

$$\textbf{P16.10} \quad y = (0.0051 \text{ m})\sin(310x - 9.30t) \text{ SI units}$$

$$v = \frac{\omega}{k} = \frac{9.30}{310} = 0.0300 \text{ m/s}$$

$$s = vt = \boxed{0.300 \text{ m in positive } x \text{ - direction}}$$

***P16.11** From $y = (12.0 \text{ cm}) \sin((1.57 \text{ rad/m})x - (31.4 \text{ rad/s})t)$

(a) The transverse velocity is $\frac{\partial y}{\partial t} = -A\omega \cos(kx - \omega t)$

Its maximum magnitude is $A\omega = 12 \text{ cm}(31.4 \text{ rad/s}) = \boxed{3.77 \text{ m/s}}$

(b) $a_y = \frac{\partial v_y}{\partial t} = \frac{\partial}{\partial t}(-A\omega \cos(kx - \omega t)) = -A\omega^2 \sin(kx - \omega t)$

The maximum value is $A\omega^2 = (0.12 \text{ m})(31.4 \text{ s}^{-1})^2 = \boxed{118 \text{ m/s}^2}$

P16.12 At time t , the phase of $y = (15.0 \text{ cm}) \cos(0.157x - 50.3t)$ at coordinate x is

$\phi = (0.157 \text{ rad/cm})x - (50.3 \text{ rad/s})t$. Since $60.0^\circ = \frac{\pi}{3} \text{ rad}$, the requirement for point B is that

$\phi_B = \phi_A \pm \frac{\pi}{3} \text{ rad}$, or (since $x_A = 0$),

$$(0.157 \text{ rad/cm})x_B - (50.3 \text{ rad/s})t = 0 - (50.3 \text{ rad/s})t \pm \frac{\pi}{3} \text{ rad}.$$

This reduces to $x_B = \frac{\pm \pi \text{ rad}}{3(0.157 \text{ rad/cm})} = \boxed{\pm 6.67 \text{ cm}}$.

P16.13 $y = 0.250 \sin(0.300x - 40.0t) \text{ m}$

Compare this with the general expression $y = A \sin(kx - \omega t)$

(a) $A = \boxed{0.250 \text{ m}}$

(b) $\omega = \boxed{40.0 \text{ rad/s}}$

(c) $k = \boxed{0.300 \text{ rad/m}}$

(d) $\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.300 \text{ rad/m}} = \boxed{20.9 \text{ m}}$

(e) $v = f\lambda = \left(\frac{\omega}{2\pi}\right)\lambda = \left(\frac{40.0 \text{ rad/s}}{2\pi}\right)(20.9 \text{ m}) = \boxed{133 \text{ m/s}}$

(f) The wave moves to the right, $\boxed{\text{in } +x \text{ direction}}$.

P16.14 (a) See figure at right.

(b) $T = \frac{2\pi}{\omega} = \frac{2\pi}{50.3} = \boxed{0.125 \text{ s}}$

This agrees with the period found in the example in the text.

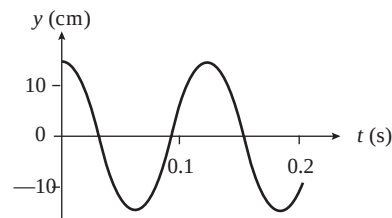


FIG. P16.14

P16.15 (a) $A = y_{\max} = 8.00 \text{ cm} = 0.0800 \text{ m}$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{(0.800 \text{ m})} = 7.85 \text{ m}^{-1}$$

$$\omega = 2\pi f = 2\pi(3.00) = 6.00\pi \text{ rad/s}$$

Therefore,

$$y = A \sin(kx + \omega t)$$

Or (where $y(0, t) = 0$ at $t = 0$)

$$\boxed{y = (0.0800) \sin(7.85x + 6\pi t) \text{ m}}$$

(b) In general,

$$y = 0.0800 \sin(7.85x + 6\pi t + \phi)$$

Assuming

$$y(x, 0) = 0 \text{ at } x = 0.100 \text{ m}$$

then we require that

$$0 = 0.0800 \sin(0.785 + \phi)$$

or

$$\phi = -0.785$$

Therefore,

$$\boxed{y = 0.0800 \sin(7.85x + 6\pi t - 0.785) \text{ m}}$$

P16.16 (a)

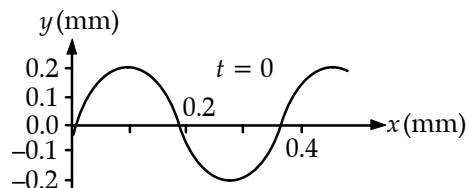


FIG. P16.16(a)

(b) $k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.350 \text{ m}} = \boxed{18.0 \text{ rad/m}}$

$$T = \frac{1}{f} = \frac{1}{12.0/\text{s}} = \boxed{0.0833 \text{ s}}$$

$$\omega = 2\pi f = 2\pi(12.0/\text{s}) = \boxed{75.4 \text{ rad/s}}$$

$$|v| = f\lambda = (12.0/\text{s})(0.350 \text{ m}) = \boxed{4.20 \text{ m/s}}$$

(c) $y = A \sin(kx + \omega t + \phi)$ specializes to

$$y = 0.200 \text{ m} \sin(18.0x/\text{m} + 75.4t/\text{s} + \phi)$$

at $x = 0$, $t = 0$ we require

$$-3.00 \times 10^{-2} \text{ m} = 0.200 \text{ m} \sin(+\phi)$$

$$\phi = -8.63^\circ = -0.151 \text{ rad}$$

so $y(x, t) = \boxed{(0.200 \text{ m}) \sin(18.0x/\text{m} + 75.4t/\text{s} - 0.151 \text{ rad})}$

P16.17 $y = (0.120 \text{ m}) \sin\left(\frac{\pi}{8}x + 4\pi t\right)$

(a) $v = \frac{dy}{dt} :$ $x = (0.120)(4\pi) \cos\left(\frac{\pi}{8}x + 4\pi t\right)$
 $v(0.200 \text{ s}, 1.60 \text{ m}) = \boxed{-1.51 \text{ m/s}}$
 $a = \frac{dv}{dt} :$ $a = (-0.120 \text{ m})(4\pi)^2 \sin\left(\frac{\pi}{8}x + 4\pi t\right)$
 $a(0.200 \text{ s}, 1.60 \text{ m}) = \boxed{0}$

(b) $k = \frac{\pi}{8} = \frac{2\pi}{\lambda} :$ $\lambda = \boxed{16.0 \text{ m}}$
 $\omega = 4\pi = \frac{2\pi}{T} :$ $T = \boxed{0.500 \text{ s}}$
 $v = \frac{\lambda}{T} = \frac{16.0 \text{ m}}{0.500 \text{ s}} = \boxed{32.0 \text{ m/s}}$

P16.18 (a) Let us write the wave function as $y(x, t) = A \sin(kx + \omega t + \phi)$
 $y(0, 0) = A \sin \phi = 0.020 \text{ m}$
 $\left. \frac{dy}{dt} \right|_{0,0} = A\omega \cos \phi = -2.00 \text{ m/s}$

Also,

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.025 \text{ s}} = 80.0 \pi/\text{s}$$

$$A^2 = x_i^2 + \left(\frac{v_i}{\omega}\right)^2 = (0.020 \text{ m})^2 + \left(\frac{2.00 \text{ m/s}}{80.0 \pi/\text{s}}\right)^2$$

$$A = \boxed{0.021 \text{ m}}$$

(b) $\frac{A \sin \phi}{A \cos \phi} = \frac{0.020 \text{ m}}{\frac{-2}{80.0\pi}} = -2.51 = \tan \phi$

Your calculator's answer $\tan^{-1}(-2.51) = -1.19 \text{ rad}$ has a negative sine and positive cosine, just the reverse of what is required. You must look beyond your calculator to find

$$\phi = \pi - 1.19 \text{ rad} = \boxed{1.95 \text{ rad}}$$

(c) $v_{y, \max} = A\omega = 0.021 \text{ m}(80.0\pi/\text{s}) = \boxed{5.41 \text{ m/s}}$

(d) $\lambda = v_x T = (30.0 \text{ m/s})(0.025 \text{ s}) = 0.750 \text{ m}$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.750 \text{ m}} = 8.38/\text{m} \qquad \omega = 80.0\pi/\text{s}$$

$$\boxed{y(x, t) = (0.021 \text{ m}) \sin(8.38x \text{ rad/m} + 80.0\pi t \text{ rad/s} + 1.95 \text{ rad})}$$

- P16.19** (a) $f = \frac{v}{\lambda} = \frac{(1.00 \text{ m/s})}{2.00 \text{ m}} = \boxed{0.500 \text{ Hz}}$
 $\omega = 2\pi f = 2\pi(0.500/\text{s}) = \boxed{3.14 \text{ rad/s}}$
- (b) $k = \frac{2\pi}{\lambda} = \frac{2\pi}{2.00 \text{ m}} = \boxed{3.14 \text{ rad/m}}$
- (c) $y = A \sin(kx - \omega t + \phi)$ becomes
 $y = \boxed{(0.100 \text{ m}) \sin(3.14x/\text{m} - 3.14t/\text{s} + 0)}$
- (d) For $x = 0$ the wave function requires
 $y = \boxed{(0.100 \text{ m}) \sin(-3.14t/\text{s})}$
- (e) $y = \boxed{(0.100 \text{ m}) \sin(4.71 \text{ rad} - 3.14t/\text{s})}$
- (f) $v_y = \frac{\partial y}{\partial t} = 0.100 \text{ m}(-3.14/\text{s}) \cos(3.14x/\text{m} - 3.14t/\text{s})$
 The cosine varies between +1 and -1, so
 $v_y \leq (0.314 \text{ m/s})$

- P16.20** (a) at $x = 2.00 \text{ m}$, $y = \boxed{(0.100 \text{ m}) \sin(1.00 \text{ rad} - 20.0t)}$
- (b) $y = (0.100 \text{ m}) \sin(0.500x - 20.0t) = A \sin(kx - \omega t)$
 so $\omega = 20.0 \text{ rad/s}$ and $f = \frac{\omega}{2\pi} = \boxed{3.18 \text{ Hz}}$

Section 16.3 The Speed of Waves on Strings

- P16.21** The down and back distance is $4.00 \text{ m} + 4.00 \text{ m} = 8.00 \text{ m}$.

The speed is then $v = \frac{d_{\text{total}}}{t} = \frac{4(8.00 \text{ m})}{0.800 \text{ s}} = 40.0 \text{ m/s} = \sqrt{\frac{T}{\mu}}$

Now, $\mu = \frac{0.200 \text{ kg}}{4.00 \text{ m}} = 5.00 \times 10^{-2} \text{ kg/m}$

So $T = \mu v^2 = (5.00 \times 10^{-2} \text{ kg/m})(40.0 \text{ m/s})^2 = \boxed{80.0 \text{ N}}$

- P16.22** The mass per unit length is: $\mu = \frac{0.0600 \text{ kg}}{5.00 \text{ m}} = 1.20 \times 10^{-2} \text{ kg/m}$.

The required tension is: $T = \mu v^2 = (0.0120 \text{ kg/m})(50.0 \text{ m/s})^2 = \boxed{30.0 \text{ N}}$.

P16.23 $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{1\,350 \text{ kg} \cdot \text{m/s}^2}{5.00 \times 10^{-3} \text{ kg/m}}} = \boxed{520 \text{ m/s}}$

P16.24 (a) $\omega = 2\pi f = 2\pi(500) = 3\,140 \text{ rad/s}$, $k = \frac{\omega}{v} = \frac{3\,140}{196} = 16.0 \text{ rad/m}$

$$y = \boxed{(2.00 \times 10^{-4} \text{ m}) \sin(16.0x - 3\,140t)}$$

(b) $v = 196 \text{ m/s} = \sqrt{\frac{T}{4.10 \times 10^{-3} \text{ kg/m}}}$

$$T = \boxed{158 \text{ N}}$$

P16.25 $T = Mg$ is the tension; $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{Mg}{\frac{m}{L}}} = \sqrt{\frac{MgL}{m}} = \frac{L}{t}$ is the wave speed.

Then, $\frac{MgL}{m} = \frac{L^2}{t^2}$

and $g = \frac{Lm}{Mt^2} = \frac{1.60 \text{ m}(4.00 \times 10^{-3} \text{ kg})}{3.00 \text{ kg}(3.61 \times 10^{-3} \text{ s})^2} = \boxed{1.64 \text{ m/s}^2}$

P16.26 $v = \sqrt{\frac{T}{\mu}}$

$$T = \mu v^2 = \rho A v^2 = \rho \pi r^2 v^2$$

$$T = (8\,920 \text{ kg/m}^3)(\pi)(7.50 \times 10^{-4} \text{ m})^2(200 \text{ m/s})^2$$

$$T = \boxed{631 \text{ N}}$$

P16.27 Since μ is constant, $\mu = \frac{T_2}{v_2^2} = \frac{T_1}{v_1^2}$ and

$$T_2 = \left(\frac{v_2}{v_1}\right)^2 T_1 = \left(\frac{30.0 \text{ m/s}}{20.0 \text{ m/s}}\right)^2 (6.00 \text{ N}) = \boxed{13.5 \text{ N}}.$$

P16.28 The period of the pendulum is $T = 2\pi \sqrt{\frac{L}{g}}$

Let F represent the tension in the string (to avoid confusion with the period) when the pendulum is vertical and stationary. The speed of waves in the string is then:

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{Mg}{\frac{m}{L}}} = \sqrt{\frac{MgL}{m}}$$

Since it might be difficult to measure L precisely, we eliminate $\sqrt{L} = \frac{T\sqrt{g}}{2\pi}$

$$\text{so } v = \sqrt{\frac{Mg}{m}} \frac{T\sqrt{g}}{2\pi} = \boxed{\frac{Tg}{2\pi} \sqrt{\frac{M}{m}}}.$$

P16.29 If the tension in the wire is T , the tensile stress is

$$\text{Stress} = \frac{T}{A} \quad \text{so} \quad T = A(\text{stress}).$$

The speed of transverse waves in the wire is

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{A(\text{Stress})}{\frac{m}{L}}} = \sqrt{\frac{\text{Stress}}{\frac{m}{AL}}} = \sqrt{\frac{\text{Stress}}{\frac{m}{\text{Volume}}}} = \sqrt{\frac{\text{Stress}}{\rho}}$$

where ρ is the density. The maximum velocity occurs when the stress is a maximum:

$$v_{\max} = \sqrt{\frac{2.70 \times 10^8 \text{ Pa}}{7860 \text{ kg/m}^3}} = \boxed{185 \text{ m/s}}.$$

P16.30 From the free-body diagram $mg = 2T \sin \theta$

$$T = \frac{mg}{2 \sin \theta}$$

The angle θ is found from $\cos \theta = \frac{\frac{3L}{8}}{\frac{L}{2}} = \frac{3}{4}$
 $\therefore \theta = 41.4^\circ$

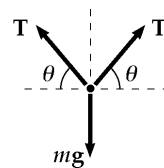


FIG. P16.30

$$(a) \quad v = \sqrt{\frac{T}{\mu}} \quad v = \sqrt{\frac{mg}{2\mu \sin 41.4^\circ}} = \left(\sqrt{\frac{9.80 \text{ m/s}^2}{2(8.00 \times 10^{-3} \text{ kg/m}) \sin 41.4^\circ}} \right) \sqrt{m}$$

$$\text{or} \quad v = \boxed{\left(30.4 \frac{\text{m/s}}{\sqrt{\text{kg}}} \right) \sqrt{m}}$$

$$(b) \quad v = 60.0 = 30.4\sqrt{m} \quad \text{and} \quad \boxed{m = 3.89 \text{ kg}}$$

P16.31 The total time is the sum of the two times.

$$\text{In each wire} \quad t = \frac{L}{v} = L \sqrt{\frac{\mu}{T}}$$

Let A represent the cross-sectional area of one wire. The mass of one wire can be written both as $m = \rho V = \rho AL$ and also as $m = \mu L$.

$$\text{Then we have} \quad \mu = \rho A = \frac{\pi \rho d^2}{4}$$

$$\text{Thus,} \quad t = L \left(\frac{\pi \rho d^2}{4T} \right)^{1/2}$$

$$\text{For copper,} \quad t = (20.0) \left[\frac{(\pi)(8920)(1.00 \times 10^{-3})^2}{(4)(150)} \right]^{1/2} = 0.137 \text{ s}$$

$$\text{For steel,} \quad t = (30.0) \left[\frac{(\pi)(7860)(1.00 \times 10^{-3})^2}{(4)(150)} \right]^{1/2} = 0.192 \text{ s}$$

$$\text{The total time is} \quad 0.137 + 0.192 = \boxed{0.329 \text{ s}}$$

P16.32 Refer to the diagrams. From the free-body diagram of point A:

$$\sum F_y = 0 \Rightarrow T_1 \sin \theta = Mg \quad \text{and} \quad \sum F_x = 0 \Rightarrow T_1 \cos \theta = T$$

Combining these equations to eliminate T_1 gives the tension in the string connecting points A and B as: $T = \frac{Mg}{\tan \theta}$.

The speed of transverse waves in this segment of string is then

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{\frac{Mg}{\tan \theta}}{\frac{m}{L}}} = \sqrt{\frac{MgL}{m \tan \theta}}$$

and the time for a pulse to travel from A to B is

$$t = \frac{L}{v} = \sqrt{\frac{mL \tan \theta}{4Mg}}.$$

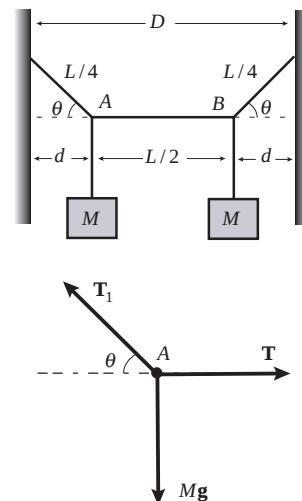


FIG. P16.32

***P16.33** (a) f has units $\text{Hz} = 1/\text{s}$, so $T = \frac{1}{f}$ has units of seconds, $\boxed{\text{s}}$. For the other T we have $T = \mu \omega^2$,

$$\text{with units } \frac{\text{kg}}{\text{m}} \frac{\text{m}^2}{\text{s}^2} = \frac{\text{kg} \cdot \text{m}}{\text{s}^2} = \boxed{\text{N}}.$$

(b) The first T is $\boxed{\text{period}}$ of time; the second is $\boxed{\text{force}}$ of tension.

Section 16.4 Reflection and Transmission

Problem 7 in Chapter 18 can be assigned with this section.

Section 16.5 Rate of Energy Transfer by Sinusoidal Waves on Strings

P16.34 $f = \frac{v}{\lambda} = \frac{30.0}{0.500} = 60.0 \text{ Hz}$ $\omega = 2\pi f = 120\pi \text{ rad/s}$

$$\mathcal{P} = \frac{1}{2} \mu \omega^2 A^2 v = \frac{1}{2} \left(\frac{0.180}{3.60} \right) (120\pi)^2 (0.100)^2 (30.0) = \boxed{1.07 \text{ kW}}$$

P16.35 Suppose that no energy is absorbed or carried down into the water. Then a fixed amount of power is spread thinner farther away from the source, spread over the circumference $2\pi r$ of an expanding circle. The power-per-width across the wave front

$$\frac{\mathcal{P}}{2\pi r}$$

is proportional to amplitude squared so amplitude is proportional to

$$\sqrt{\frac{\mathcal{P}}{2\pi r}}.$$

P16.36 $T = \text{constant}; v = \sqrt{\frac{T}{\mu}}; \mathcal{P} = \frac{1}{2} \mu \omega^2 A^2 v$

- (a) If L is doubled, v remains constant and $\mathcal{P}$ is constant.
- (b) If A is doubled and ω is halved, $\mathcal{P} \propto \omega^2 A^2$ remains constant.
- (c) If λ and A are doubled, the product $\omega^2 A^2 \propto \frac{A^2}{\lambda^2}$ remains constant, so $\mathcal{P}$ remains constant.
- (d) If L and λ are halved, then $\omega^2 \propto \frac{1}{\lambda^2}$ is quadrupled, so $\mathcal{P}$ is quadrupled.
(Changing L doesn't affect $\mathcal{P}$).

P16.37 $A = 5.00 \times 10^{-2} \text{ m}$ $\mu = 4.00 \times 10^{-2} \text{ kg/m}$ $\mathcal{P} = 300 \text{ W}$ $T = 100 \text{ N}$

Therefore, $v = \sqrt{\frac{T}{\mu}} = 50.0 \text{ m/s}$

$$\mathcal{P} = \frac{1}{2} \mu \omega^2 A^2 v : \quad \omega^2 = \frac{2\mathcal{P}}{\mu A^2 v} = \frac{2(300)}{(4.00 \times 10^{-2})(5.00 \times 10^{-2})^2 (50.0)}$$

$$\omega = 346 \text{ rad/s}$$

$$f = \frac{\omega}{2\pi} = 55.1 \text{ Hz}$$

P16.38 $\mu = 30.0 \text{ g/m} = 30.0 \times 10^{-3} \text{ kg/m}$
 $\lambda = 1.50 \text{ m}$
 $f = 50.0 \text{ Hz} : \quad \omega = 2\pi f = 314 \text{ s}^{-1}$
 $2A = 0.150 \text{ m} : \quad A = 7.50 \times 10^{-2} \text{ m}$

(a) $y = A \sin\left(\frac{2\pi}{\lambda} x - \omega t\right)$

$$y = (7.50 \times 10^{-2}) \sin(4.19x - 314t)$$

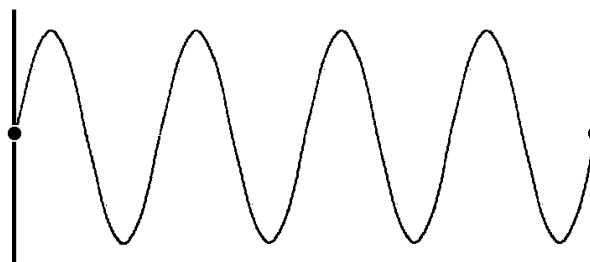


FIG. P16.38

(b) $\mathcal{P} = \frac{1}{2} \mu \omega^2 A^2 v = \frac{1}{2} (30.0 \times 10^{-3}) (314)^2 (7.50 \times 10^{-2})^2 \left(\frac{314}{4.19}\right) \text{ W} = 625 \text{ W}$

P16.39 (a) $v = f\lambda = \frac{\omega}{2\pi} \frac{2\pi}{k} = \frac{\omega}{k} = \frac{50.0}{0.800} \text{ m/s} = 62.5 \text{ m/s}$

(b) $\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.800} \text{ m} = 7.85 \text{ m}$

(c) $f = \frac{50.0}{2\pi} = 7.96 \text{ Hz}$

(d) $\mathcal{P} = \frac{1}{2} \mu \omega^2 A^2 v = \frac{1}{2} (12.0 \times 10^{-3}) (50.0)^2 (0.150)^2 (62.5) \text{ W} = 21.1 \text{ W}$

***P16.40** Comparing $y = 0.35 \sin\left(10\pi t - 3\pi x + \frac{\pi}{4}\right)$ with $y = A \sin(kx - \omega t + \phi) = A \sin(\omega t - kx - \phi + \pi)$ we have

$$k = \frac{3\pi}{m}, \quad \omega = 10\pi/\text{s}, \quad A = 0.35 \text{ m}. \quad \text{Then } v = f\lambda = 2\pi f \frac{\lambda}{2\pi} = \frac{\omega}{k} = \frac{10\pi/\text{s}}{3\pi/\text{m}} = 3.33 \text{ m/s}.$$

(a) The rate of energy transport is

$$\mathcal{P} = \frac{1}{2} \mu \omega^2 A^2 v = \frac{1}{2} (75 \times 10^{-3} \text{ kg/m}) (10\pi/\text{s})^2 (0.35 \text{ m})^2 3.33 \text{ m/s} = \boxed{15.1 \text{ W}}.$$

(b) The energy per cycle is

$$E_\lambda = \mathcal{P} T = \frac{1}{2} \mu \omega^2 A^2 \lambda = \frac{1}{2} (75 \times 10^{-3} \text{ kg/m}) (10\pi/\text{s})^2 (0.35 \text{ m})^2 \frac{2\pi \text{ m}}{3\pi} = \boxed{3.02 \text{ J}}.$$

P16.41 Originally,

$$\begin{aligned} \mathcal{P}_0 &= \frac{1}{2} \mu \omega^2 A^2 v \\ \mathcal{P}_0 &= \frac{1}{2} \mu \omega^2 A^2 \sqrt{\frac{T}{\mu}} \\ \mathcal{P}_0 &= \frac{1}{2} \omega^2 A^2 \sqrt{T\mu} \end{aligned}$$

The doubled string will have doubled mass-per-length. Presuming that we hold tension constant, it can carry power larger by $\sqrt{2}$ times.

$$\boxed{\sqrt{2}\mathcal{P}_0} = \frac{1}{2} \omega^2 A^2 \sqrt{T2\mu}$$

***P16.42** As for a strong wave, the rate of energy transfer is proportional to the square of the amplitude and to the speed. We write $\mathcal{P} = FvA^2$ where F is some constant. With no absorption of energy,

$$\begin{aligned} Fv_{\text{bedrock}} A_{\text{bedrock}}^2 &= Fv_{\text{mudfill}} A_{\text{mudfill}}^2 \\ \sqrt{\frac{v_{\text{bedrock}}}{v_{\text{mudfill}}}} &= \frac{A_{\text{mudfill}}}{A_{\text{bedrock}}} = \sqrt{\frac{25v_{\text{mudfill}}}{v_{\text{mudfill}}}} = 5 \end{aligned}$$

The amplitude increases by 5.00 times.

Section 16.6 The Linear Wave Equation

- P16.43** (a) $A = (7.00 + 3.00)4.00$ yields $A = 40.0$
- (b) In order for two vectors to be equal, they must have the same magnitude and the same direction in three-dimensional space. All of their components must be equal. Thus, $7.00\hat{i} + 0\hat{j} + 3.00\hat{k} = A\hat{i} + B\hat{j} + C\hat{k}$ requires $A = 7.00$, $B = 0$, and $C = 3.00$.
- (c) In order for two functions to be identically equal, they must be equal for every value of every variable. They must have the same graphs. In

$$A + B\cos(Cx + Dt + E) = 0 + 7.00 \text{ mm} \cos(3.00x + 4.00t + 2.00),$$

the equality of average values requires that $A = 0$. The equality of maximum values requires $B = 7.00 \text{ mm}$. The equality for the wavelength or periodicity as a function of x requires $C = 3.00 \text{ rad/m}$. The equality of period requires $D = 4.00 \text{ rad/s}$, and the equality of zero-crossings requires $E = 2.00 \text{ rad}$.

***P16.44** The linear wave equation is $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$

If $y = e^{b(x-vt)}$

then $\frac{\partial y}{\partial t} = -bve^{b(x-vt)}$ and $\frac{\partial y}{\partial x} = be^{b(x-vt)}$

$$\frac{\partial^2 y}{\partial t^2} = b^2 v^2 e^{b(x-vt)} \text{ and } \frac{\partial^2 y}{\partial x^2} = b^2 e^{b(x-vt)}$$

Therefore, $\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$, demonstrating that $e^{b(x-vt)}$ is a solution

P16.45 The linear wave equation is $\frac{1}{v^2} \frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2}$

To show that $y = \ln[b(x - vt)]$ is a solution, we find its first and second derivatives with respect to x and t and substitute into the equation.

$$\frac{\partial y}{\partial t} = \frac{1}{b(x-vt)}(-bv) \quad \frac{\partial^2 y}{\partial t^2} = \frac{-1(-bv)^2}{b^2(x-vt)^2} = -\frac{v^2}{(x-vt)^2}$$

$$\frac{\partial y}{\partial x} = [b(x-vt)]^{-1}b \quad \frac{\partial^2 y}{\partial x^2} = -\frac{b}{b^2(x-vt)^2} = -\frac{1}{b(x-vt)^2}$$

Then $\frac{1}{v^2} \frac{\partial^2 y}{\partial t^2} = \frac{1}{v^2} \frac{(-v^2)}{(x-vt)^2} = -\frac{1}{(x-vt)^2} = \frac{\partial^2 y}{\partial x^2}$ so the given wave function is a solution.

P16.46 (a) From $y = x^2 + v^2 t^2$,

$$\begin{aligned} \text{evaluate } \frac{\partial y}{\partial x} &= 2x & \frac{\partial^2 y}{\partial x^2} &= 2 \\ \frac{\partial y}{\partial t} &= v^2 2t & \frac{\partial^2 y}{\partial t^2} &= 2v^2 \end{aligned}$$

$$\text{Does } \frac{\partial^2 y}{\partial t^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial x^2} ?$$

By substitution: $2 = \frac{1}{v^2} 2v^2$ and this is true, so the wave function does satisfy the wave equation.

$$\begin{aligned} \text{(b) Note } \frac{1}{2}(x+vt)^2 + \frac{1}{2}(x-vt)^2 &= \frac{1}{2}x^2 + xvt + \frac{1}{2}v^2t^2 + \frac{1}{2}x^2 - xvt + \frac{1}{2}v^2t^2 \\ &= x^2 + v^2t^2 \text{ as required.} \end{aligned}$$

$$\text{So } \boxed{f(x+vt) = \frac{1}{2}(x+vt)^2} \text{ and } \boxed{g(x-vt) = \frac{1}{2}(x-vt)^2}.$$

(c) $y = \sin x \cos vt$ makes

$$\begin{aligned} \frac{\partial y}{\partial x} &= \cos x \cos vt & \frac{\partial^2 y}{\partial x^2} &= -\sin x \cos vt \\ \frac{\partial y}{\partial t} &= -v \sin x \sin vt & \frac{\partial^2 y}{\partial t^2} &= -v^2 \sin x \cos vt \end{aligned}$$

$$\text{Then } \frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$$

becomes $-\sin x \cos vt = \frac{-1}{v^2} v^2 \sin x \cos vt$ which is true as required.

$$\text{Note } \sin(x+vt) = \sin x \cos vt + \cos x \sin vt$$

$$\sin(x-vt) = \sin x \cos vt - \cos x \sin vt.$$

So $\sin x \cos vt = f(x+vt) + g(x-vt)$ with

$$\boxed{f(x+vt) = \frac{1}{2} \sin(x+vt)} \quad \text{and} \quad \boxed{g(x-vt) = \frac{1}{2} \sin(x-vt)}.$$

Additional Problems

P16.47 Assume a typical distance between adjacent people ~ 1 m.

$$\text{Then the wave speed is } v = \frac{\Delta x}{\Delta t} \sim \frac{1 \text{ m}}{0.1 \text{ s}} \sim 10 \text{ m/s}$$

Model the stadium as a circle with a radius of order 100 m. Then, the time for one circuit around the stadium is

$$T = \frac{2\pi r}{v} \sim \frac{2\pi(10^2)}{10 \text{ m/s}} = 63 \text{ s } \boxed{\sim 1 \text{ min}}.$$

P16.48 Compare the given wave function $y = 4.00 \sin(2.00x - 3.00t)$ cm to the general form $y = A \sin(kx - \omega t)$ to find

(a) amplitude $A = 4.00 \text{ cm} = \boxed{0.0400 \text{ m}}$

(b) $k = \frac{2\pi}{\lambda} = 2.00 \text{ cm}^{-1}$ and $\lambda = \pi \text{ cm} = \boxed{0.0314 \text{ m}}$

(c) $\omega = 2\pi f = 3.00 \text{ s}^{-1}$ and $f = \boxed{0.477 \text{ Hz}}$

(d) $T = \frac{1}{f} = \boxed{2.09 \text{ s}}$

(e) The minus sign indicates that the wave is traveling in the $\boxed{\text{positive } x\text{-direction}}$.

P16.49 (a) Let $u = 10\pi t - 3\pi x + \frac{\pi}{4}$ $\frac{du}{dt} = 10\pi - 3\pi \frac{dx}{dt} = 0$ at a point of constant phase

$$\frac{dx}{dt} = \frac{10}{3} = \boxed{3.33 \text{ m/s}}$$

The velocity is in the $\boxed{\text{positive } x\text{-direction}}$.

(b) $y(0.100, 0) = (0.350 \text{ m}) \sin\left(-0.300\pi + \frac{\pi}{4}\right) = -0.0548 \text{ m} = \boxed{-5.48 \text{ cm}}$

(c) $k = \frac{2\pi}{\lambda} = 3\pi : \lambda = \boxed{0.667 \text{ m}}$ $\omega = 2\pi f = 10\pi : f = \boxed{5.00 \text{ Hz}}$

(d) $v_y = \frac{\partial y}{\partial t} = (0.350)(10\pi) \cos\left(10\pi t - 3\pi x + \frac{\pi}{4}\right)$ $v_{y, \max} = (10\pi)(0.350) = \boxed{11.0 \text{ m/s}}$

***P16.50** (a) $0.175 \text{ m} = (0.350 \text{ m}) \sin[(99.6 \text{ rad/s})t]$

$$\therefore \sin[(99.6 \text{ rad/s})t] = 0.5$$

The smallest two angles for which the sine function is 0.5 are 30° and 150° , i.e., 0.5236 rad and 2.618 rad .

$$(99.6 \text{ rad/s})t_1 = 0.5236 \text{ rad}, \text{ thus } t_1 = 5.26 \text{ ms}$$

$$(99.6 \text{ rad/s})t_2 = 2.618 \text{ rad}, \text{ thus } t_2 = 26.3 \text{ ms}$$

$$\Delta t \equiv t_2 - t_1 = 26.3 \text{ ms} - 5.26 \text{ ms} = \boxed{21.0 \text{ ms}}$$

(b) Distance traveled by the wave $= \left(\frac{\omega}{k}\right) \Delta t = \left(\frac{99.6 \text{ rad/s}}{1.25 \text{ rad/m}}\right) (21.0 \times 10^{-3} \text{ s}) = \boxed{1.68 \text{ m}}$.

P16.51 The equation $v = \lambda f$ is a special case of

speed = (cycle length)(repetition rate).

$$\text{Thus, } v = (19.0 \times 10^{-3} \text{ m/frame})(24.0 \text{ frames/s}) = \boxed{0.456 \text{ m/s}}.$$

P16.52 Assuming the incline to be frictionless and taking the positive x -direction to be up the incline:

$$\sum F_x = T - Mg \sin \theta = 0 \text{ or the tension in the string is } T = Mg \sin \theta$$

The speed of transverse waves in the string is then
$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{Mg \sin \theta}{\frac{m}{L}}} = \sqrt{\frac{MgL \sin \theta}{m}}$$

The time interval for a pulse to travel the string's length is
$$\Delta t = \frac{L}{v} = L \sqrt{\frac{m}{MgL \sin \theta}} = \boxed{\sqrt{\frac{mL}{Mg \sin \theta}}}$$

P16.53 Energy is conserved as the block moves down distance x :

$$\begin{aligned} (K + U_g + U_s)_{\text{top}} + \Delta E &= (K + U_g + U_s)_{\text{bottom}} \\ 0 + Mgx + 0 + 0 &= 0 + 0 + \frac{1}{2}kx^2 \\ x &= \frac{2Mg}{k} \end{aligned}$$

(a) $T = kx = 2Mg = 2(2.00 \text{ kg})(9.80 \text{ m/s}^2) = \boxed{39.2 \text{ N}}$

(b)
$$L = L_0 + x = L_0 + \frac{2Mg}{k}$$
$$L = 0.500 \text{ m} + \frac{39.2 \text{ N}}{100 \text{ N/m}} = \boxed{0.892 \text{ m}}$$

(c)
$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{TL}{m}}$$
$$v = \sqrt{\frac{39.2 \text{ N} \times 0.892 \text{ m}}{5.0 \times 10^{-3} \text{ kg}}}$$
$$v = \boxed{83.6 \text{ m/s}}$$

P16.54 $Mgx = \frac{1}{2}kx^2$

(a) $T = kx = \boxed{2Mg}$

(b)
$$L = L_0 + x = \boxed{L_0 + \frac{2Mg}{k}}$$

(c)
$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{TL}{m}} = \boxed{\sqrt{\frac{2Mg}{m} \left(L_0 + \frac{2Mg}{k} \right)}}$$

P16.55 (a) $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{80.0 \text{ N}}{(5.00 \times 10^{-3} \text{ kg}/2.00 \text{ m})}} = \boxed{179 \text{ m/s}}$

(b) From Equation 16.21, $\mathcal{P} = \frac{1}{2} \mu v \omega^2 A^2$ and $\omega = 2\pi \left(\frac{v}{\lambda} \right)$

$$\mathcal{P} = \frac{1}{2} \mu v A^2 \left(\frac{2\pi v}{\lambda} \right)^2 = \frac{2\pi^2 \mu A^2 v^3}{\lambda^2}$$

$$\mathcal{P} = \frac{2\pi^2 \left(\frac{5.00 \times 10^{-3} \text{ kg}}{2.00 \text{ m}} \right) (0.0400 \text{ m})^2 (179 \text{ m/s})^3}{(0.160 \text{ m})^2}$$

$$\mathcal{P} = 1.77 \times 10^4 \text{ W} = \boxed{17.7 \text{ kW}}$$

P16.56 $v = \sqrt{\frac{T}{\mu}}$ and in this case $T = mg$; therefore, $m = \frac{\mu v^2}{g}$.

Now $v = f\lambda$ implies $v = \frac{\omega}{k}$ so that

$$m = \frac{\mu \left(\frac{\omega}{k} \right)^2}{g} = \frac{0.250 \text{ kg/m}}{9.80 \text{ m/s}^2} \left[\frac{18\pi \text{ s}^{-1}}{0.750\pi \text{ m}^{-1}} \right]^2 = \boxed{14.7 \text{ kg}}.$$

***P16.57** Let M = mass of block, m = mass of string. For the block, $\sum F = ma$ implies $T = \frac{mv_b^2}{r} = m\omega^2 r$. The speed of a wave on the string is then

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{M\omega^2 r}{\frac{m}{r}}} = r\omega \sqrt{\frac{M}{m}}$$

$$t = \frac{r}{v} = \frac{1}{\omega} \sqrt{\frac{m}{M}}$$

$$\theta = \omega t = \sqrt{\frac{m}{M}} = \sqrt{\frac{0.0032 \text{ kg}}{0.450 \text{ kg}}} = \boxed{0.0843 \text{ rad}}$$

P16.58 (a) $\mu = \frac{dm}{dL} = \rho A \frac{dx}{dx} = \rho A$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\rho A}} = \sqrt{\frac{T}{\rho(ax+b)}} = \sqrt{\frac{T}{\rho(10^{-3}x + 10^{-2})\text{cm}^2}}$$

With all SI units, $v = \sqrt{\frac{T}{\rho(10^{-3}x + 10^{-2})10^{-4}}} \text{ m/s}$

(b) $v|_{x=0} = \sqrt{\frac{24.0}{[(2700)(0 + 10^{-2})(10^{-4})]}} = \boxed{94.3 \text{ m/s}}$

$$v|_{x=10.0} = \sqrt{\frac{24.0}{[(2700)(10^{-2} + 10^{-2})(10^{-4})]}} = \boxed{66.7 \text{ m/s}}$$

P16.59 $v = \sqrt{\frac{T}{\mu}}$ where $T = \mu x g$, the weight of a length x , of rope.

Therefore, $v = \sqrt{g x}$

But $v = \frac{dx}{dt}$, so that $dt = \frac{dx}{\sqrt{g x}}$

and
$$t = \int_0^L \frac{dx}{\sqrt{g x}} = \frac{1}{\sqrt{g}} \left. \frac{\sqrt{x}}{\frac{1}{2}} \right|_0^L = \boxed{2\sqrt{\frac{L}{g}}}$$

P16.60 At distance x from the bottom, the tension is $T = \left(\frac{m x g}{L}\right) + M g$, so the wave speed is:

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{TL}{m}} = \sqrt{xg + \left(\frac{MgL}{m}\right)} = \frac{dx}{dt}.$$

(a) Then $t = \int_0^L dt = \int_0^L \left[xg + \left(\frac{MgL}{m}\right) \right]^{-1/2} dx$ $t = \frac{1}{g} \left[xg + \left(\frac{MgL}{m}\right) \right]^{1/2} \Big|_{x=0}^{x=L}$

$$t = \frac{2}{g} \left[\left(Lg + \frac{MgL}{m} \right)^{1/2} - \left(\frac{MgL}{m} \right)^{1/2} \right]$$

$$\boxed{t = 2\sqrt{\frac{L}{g}} \left(\frac{\sqrt{m+M} - \sqrt{M}}{\sqrt{m}} \right)}$$

(b) When $M = 0$, as in the previous problem, $t = 2\sqrt{\frac{L}{g}} \left(\frac{\sqrt{m} - 0}{\sqrt{m}} \right) = \boxed{2\sqrt{\frac{L}{g}}}$

(c) As $m \rightarrow 0$ we expand $\sqrt{m+M} = \sqrt{M} \left(1 + \frac{m}{M} \right)^{1/2} = \sqrt{M} \left(1 + \frac{1}{2} \frac{m}{M} - \frac{1}{8} \frac{m^2}{M^2} + \dots \right)$

to obtain
$$t = 2\sqrt{\frac{L}{g}} \left(\frac{\sqrt{M} + \frac{1}{2} \left(m/\sqrt{M} \right) - \frac{1}{8} \left(m^2/M^{3/2} \right) + \dots - \sqrt{M}}{\sqrt{m}} \right)$$

$$t \approx 2\sqrt{\frac{L}{g}} \left(\frac{1}{2} \sqrt{\frac{m}{M}} \right) = \boxed{\sqrt{\frac{mL}{Mg}}}$$

P16.61 (a) The speed in the lower half of a rope of length L is the same function of distance (from the bottom end) as the speed along the entire length of a rope of length $\left(\frac{L}{2}\right)$.

Thus, the time required $= 2\sqrt{\frac{L'}{g}}$ with $L' = \frac{L}{2}$

and the time required $= 2\sqrt{\frac{L}{2g}} = \boxed{0.707 \left(2\sqrt{\frac{L}{g}} \right)}$.

It takes the pulse more than 70% of the total time to cover 50% of the distance.

(b) By the same reasoning applied in part (a), the distance climbed in τ is given by $d = \frac{g\tau^2}{4}$.

For $\tau = \frac{t}{2} = \sqrt{\frac{L}{g}}$, we find the distance climbed $= \boxed{\frac{L}{4}}$.

In half the total trip time, the pulse has climbed $\frac{1}{4}$ of the total length.

P16.62 (a) $v = \frac{\omega}{k} = \frac{15.0}{3.00} = \boxed{5.00 \text{ m/s in positive } x\text{-direction}}$

(b) $v = \frac{15.0}{3.00} = \boxed{5.00 \text{ m/s in negative } x\text{-direction}}$

(c) $v = \frac{15.0}{2.00} = \boxed{7.50 \text{ m/s in negative } x\text{-direction}}$

(d) $v = \frac{12.0}{\frac{1}{2}} = \boxed{24.0 \text{ m/s in positive } x\text{-direction}}$

P16.63 Young's modulus for the wire may be written as $Y = \frac{T/A}{\Delta L/L}$, where T is the tension maintained in the wire and ΔL is the elongation produced by this tension. Also, the mass density of the wire may be expressed as $\rho = \frac{\mu}{A}$.

The speed of transverse waves in the wire is then

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T/A}{\mu/A}} = \sqrt{\frac{Y(\Delta L/L)}{\rho}}$$

and the strain in the wire is $\frac{\Delta L}{L} = \frac{\rho v^2}{Y}$.

If the wire is aluminum and $v = 100 \text{ m/s}$, the strain is

$$\frac{\Delta L}{L} = \frac{(2.70 \times 10^3 \text{ kg/m}^3)(100 \text{ m/s})^2}{7.00 \times 10^{10} \text{ N/m}^2} = \boxed{3.86 \times 10^{-4}}$$

- *P16.64** (a) Consider a short section of chain at the top of the loop. A free-body diagram is shown. Its length is $s = R(2\theta)$ and its mass is $\mu R 2\theta$. In the frame of reference of the center of the loop, Newton's second law is

$$\sum F_y = ma_y \quad 2T \sin \theta \text{ down} = \frac{mv_0^2}{R} \text{ down} = \frac{\mu R 2\theta v_0^2}{R}$$

For a very short section, $\sin \theta = \theta$ and $\boxed{T = \mu v_0^2}$.

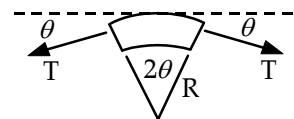


FIG. P16.64(a)

- (b) The wave speed is $v = \sqrt{\frac{T}{\mu}} = \boxed{v_0}$.

- (c) In the frame of reference of the center of the loop, each pulse moves with equal speed clockwise and counterclockwise.

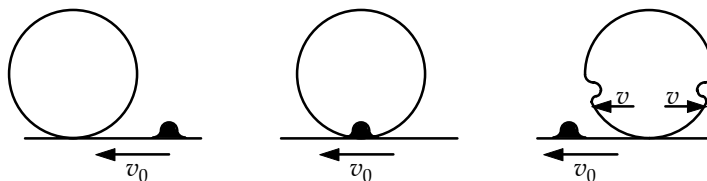


FIG. P16.64(c-1)

continued on next page

In the frame of reference of the ground, once pulse moves backward at speed $v_0 + v = 2v_0$ and the other forward at $v_0 - v = 0$. The one pulse makes two revolutions while the loop makes one revolution and the other pulse does not move around the loop. If it is generated at the six-o'clock position, it will stay at the six-o'clock position.

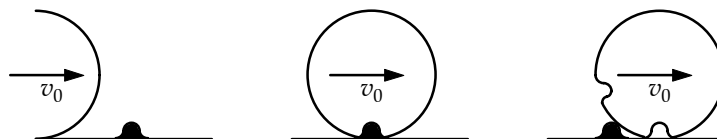


FIG. P16.64(c-2)

- P16.65** (a) Assume the spring is originally stationary throughout, extended to have a length L much greater than its equilibrium length. We start moving one end forward with the speed v at which a wave propagates on the spring. In this way we create a single pulse of compression that moves down the length of the spring. For an increment of spring with length dx and mass dm , just as the pulse swallows it up, $\sum F = ma$

becomes $kdx = adm$ or $\frac{k}{\frac{dm}{dx}} = a$.

But $\frac{dm}{dx} = \mu$ so $a = \frac{k}{\mu}$.

Also, $a = \frac{dv}{dt} = \frac{v}{t}$ when $v_i = 0$. But $L = vt$, so $a = \frac{v^2}{L}$.

Equating the two expressions for a , we have $\frac{k}{\mu} = \frac{v^2}{L}$ or $v = \sqrt{\frac{kL}{\mu}}$.

- (b) Using the expression from part (a) $v = \sqrt{\frac{kL}{\mu}} = \sqrt{\frac{kL^2}{m}} = \sqrt{\frac{(100 \text{ N/m})(2.00 \text{ m})^2}{0.400 \text{ kg}}} = \boxed{31.6 \text{ m/s}}$.

- P16.66** (a) $v = \left(\frac{T}{\mu}\right)^{1/2} = \left(\frac{2T_0}{\mu_0}\right)^{1/2} = \boxed{v_0\sqrt{2}}$ where $v_0 \equiv \left(\frac{T_0}{\mu_0}\right)^{1/2}$

$$v' = \left(\frac{T}{\mu'}\right)^{1/2} = \left(\frac{2T_0}{3\mu_0}\right)^{1/2} = \boxed{v_0\sqrt{\frac{2}{3}}}$$

- (b) $\Delta t_{\text{left}} = \frac{\frac{L}{2}}{v} = \frac{L}{2v_0\sqrt{2}} = \frac{\Delta t_0}{2\sqrt{2}} = 0.354\Delta t_0$ where $\Delta t_0 \equiv \frac{L}{v_0}$

$$\Delta t_{\text{right}} = \frac{\frac{L}{2}}{v'} = \frac{L}{2v_0\sqrt{\frac{2}{3}}} = \frac{\Delta t_0}{2\sqrt{\frac{2}{3}}} = 0.612\Delta t_0$$

$$\Delta t_{\text{left}} + \Delta t_{\text{right}} = \boxed{0.966\Delta t_0}$$

P16.67 (a) $\mathcal{P}(x) = \frac{1}{2} \mu \omega^2 A^2 v = \frac{1}{2} \mu \omega^2 A_0^2 e^{-2bx} \left(\frac{\omega}{k} \right) = \boxed{\frac{\mu \omega^3}{2k} A_0^2 e^{-2bx}}$

(b) $\mathcal{P}(0) = \boxed{\frac{\mu \omega^3}{2k} A_0^2}$

(c) $\frac{\mathcal{P}(x)}{\mathcal{P}(0)} = \boxed{e^{-2bx}}$

P16.68 $v = \frac{4\,450 \text{ km}}{9.50 \text{ h}} = 468 \text{ km/h} = \boxed{130 \text{ m/s}}$

$$\bar{d} = \frac{v^2}{g} = \frac{(130 \text{ m/s})^2}{(9.80 \text{ m/s}^2)} = \boxed{1\,730 \text{ m}}$$

***P16.69** (a) $\mu(x)$ is a linear function, so it is of the form $\mu(x) = mx + b$
 To have $\mu(0) = \mu_0$ we require $b = \mu_0$. Then $\mu(L) = \mu_L = mL + \mu_0$
 so $m = \frac{\mu_L - \mu_0}{L}$

Then

$$\mu(x) = \frac{(\mu_L - \mu_0)x}{L} + \mu_0$$

(b) From $v = \frac{dx}{dt}$, the time required to move from x to $x + dx$ is $\frac{dx}{v}$. The time required to move from 0 to L is

$$\begin{aligned} \Delta t &= \int_0^L \frac{dx}{v} = \int_0^L \frac{dx}{\sqrt{\frac{T}{\mu}}} = \frac{1}{\sqrt{T}} \int_0^L \sqrt{\mu(x)} dx \\ \Delta t &= \frac{1}{\sqrt{T}} \int_0^L \left(\frac{(\mu_L - \mu_0)x}{L} + \mu_0 \right)^{1/2} \left(\frac{\mu_L - \mu_0}{L} \right) dx \left(\frac{L}{\mu_L - \mu_0} \right) \\ \Delta t &= \frac{1}{\sqrt{T}} \left(\frac{L}{\mu_L - \mu_0} \right) \left(\frac{(\mu_L - \mu_0)x}{L} + \mu_0 \right)^{3/2} \frac{1}{\frac{3}{2}} \bigg|_0^L \\ \Delta t &= \frac{2L}{3\sqrt{T}(\mu_L - \mu_0)} (\mu_L^{3/2} - \mu_0^{3/2}) \\ \Delta t &= \frac{2L(\sqrt{\mu_L} - \sqrt{\mu_0})(\mu_L + \sqrt{\mu_L\mu_0} + \mu_0)}{3\sqrt{T}(\sqrt{\mu_L} - \sqrt{\mu_0})(\sqrt{\mu_L} + \sqrt{\mu_0})} \\ \Delta t &= \frac{2L}{3\sqrt{T}} \left(\frac{\mu_L + \sqrt{\mu_L\mu_0} + \mu_0}{\sqrt{\mu_L} + \sqrt{\mu_0}} \right) \end{aligned}$$

ANSWERS TO EVEN PROBLEMS

- P16.2** see the solution
- P16.4** (a) the P wave; (b) 665 s
- P16.6** 0.800 m/s
- P16.8** 2.40 m/s
- P16.10** 0.300 m in the positive x -direction
- P16.12** ± 6.67 cm
- P16.14** (a) see the solution; (b) 0.125 s; in agreement with the example
- P16.16** (a) see the solution; (b) 18.0/m; 83.3 ms; 75.4 rad/s; 4.20 m/s; (c) $(0.2 \text{ m})\sin(18x + 75.4t - 0.151)$
- P16.18** (a) 0.021 5 m; (b) 1.95 rad; (c) 5.41 m/s; (d) $y(x, t) = (0.021 5 \text{ m})\sin(8.38x + 80.0\pi t + 1.95)$
- P16.20** (a) see the solution; (b) 3.18 Hz
- P16.22** 30.0 N
- P16.24** (a) $y = (0.2 \text{ mm})\sin(16x - 3 140t)$; (b) 158 N
- P16.26** 631 N
- P16.28** $v = \frac{Tg}{2\pi} \sqrt{\frac{M}{m}}$
- P16.30** (a) $v = \left(30.4 \frac{\text{m}}{\text{s} \cdot \sqrt{\text{kg}}} \right) \sqrt{m}$; (b) 3.89 kg
- P16.32** $\sqrt{\frac{mL \tan \theta}{4Mg}}$
- P16.34** 1.07 kW
- P16.36** (a), (b), (c) $\mathcal{P}$ is constant; (d) $\mathcal{P}$ is quadrupled
- P16.38** (a) $y = (0.075 0)\sin(4.19x - 314t)$; (b) 625 W
- P16.40** (a) 15.1 W; (b) 3.02 J
- P16.42** The amplitude increases by 5.00 times
- P16.44** see the solution
- P16.46** (a) see the solution; (b) $\frac{1}{2}(x + vt)^2 + \frac{1}{2}(x - vt)^2$; (c) $\frac{1}{2}\sin(x + vt) + \frac{1}{2}\sin(x - vt)$
- P16.48** (a) 0.040 0 m; (b) 0.031 4 m; (c) 0.477 Hz; (d) 2.09 s; (e) positive x -direction
- P16.50** (a) 21.0 ms; (b) 1.68 m
- P16.52** $\Delta t = \sqrt{\frac{mL}{Mg \sin \theta}}$
- P16.54** (a) $2Mg$; (b) $L_0 + \frac{2Mg}{k}$; (c) $\sqrt{\frac{2Mg}{m} \left(L_0 + \frac{2Mg}{k} \right)}$
- P16.56** 14.7 kg
- P16.58** (a) $v = \sqrt{\frac{T}{\rho(10^{-7}x + 10^{-6})}}$ in SI units; (b) 94.3 m/s; 66.7 m/s
- P16.60** see the solution
- P16.62** (a) $5.00\hat{i}$ m/s; (b) $-5.00\hat{i}$ m/s; (c) $-7.50\hat{i}$ m/s; (d) $24.0\hat{i}$ m/s
- P16.64** (a) μv_0^2 ; (b) v_0 ; (c) One travels 2 rev and the other does not move around the loop.

P16.66 (a) $v = \left(\frac{2T_0}{\mu_0} \right)^{1/2} = v_0 \sqrt{2};$

$v' = \left(\frac{2T_0}{3\mu_0} \right)^{1/2} = v_0 \sqrt{\frac{2}{3}}; \text{ (b) } 0.966\Delta t_0$

P16.68 130 m/s; 1.73 km